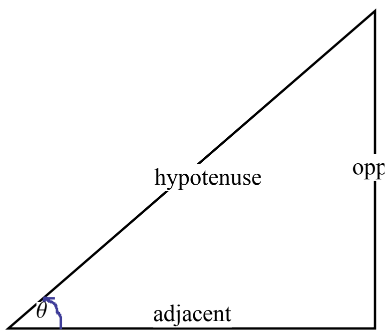
Business PreCalculus MATH 1643 Section 004, Spring 2014
Lesson 29: Right Triangle Trigonometry

Consider a right triangle with one of its acute angles labeled θ. Let $a =$ length of the side opposite to θ, $b =$ length of the side adjacent to θ, and $c =$ length of the hypotenuse. We can form six ratios with



the lengths of these sides: $\frac{a}{c}, \frac{b}{c}, \frac{a}{b}, \frac{b}{a}, \frac{c}{b}, \frac{c}{a}$.

Definition 1. **Right Triangle Definitions of the Trigonometric Functions:**

$$sin\theta = \frac{opposite}{hypotenuse} \qquad csc\theta = \frac{hypotenuse}{opposite}$$

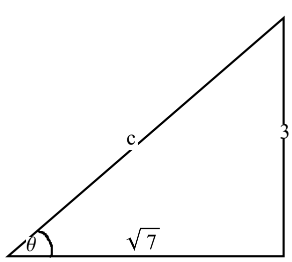
$$cos\theta = \frac{adjacent}{hypotenuse} \qquad sec\theta = \frac{hypotenuse}{adjacent}$$

$$tan\theta = \frac{opposite}{adjacent} \qquad cot\theta = \frac{adjacent}{opposite}$$

Definition 2. **Pythagorean Theorem:** *The length of the hypotenuse of a right angle triangle is related to the length of the other sides as follows:*

$$(opposite)^2 + (adjacent)^2 = (hypotenuse)^2$$
$$a^2 + b^2 = c^2.$$

Example 1. *Find the values for the six trigonometric functions of the angle θ.*



Solution: *To find the values for the six trigonometric functions, we must first find the value of c, the length of the hypotenuse.*

$$a^2 + b^2 = c^2$$

$$(3)^2 + (\sqrt{7})^2 = c^2$$
$$16 = c^2.$$

Then $c = 4$*. Hence, using the above definition we have,*

$$sin\theta = \frac{3}{4} \qquad csc\theta = \frac{4}{3}$$

$$cos\theta = \frac{\sqrt{7}}{4} \qquad sec\theta = \frac{4}{\sqrt{7}}$$

$$tan\theta = \frac{3}{\sqrt{7}} \qquad cot\theta = \frac{\sqrt{7}}{3}$$

Definition 3. **Quotient Identities:** *We can relate the trigonometric functions* ***tangent*** *and* ***cotangent*** *with the* ***sine and cosine****:*

$$tan\theta = \frac{sin\theta}{cos\theta} \qquad cot\theta = \frac{cos\theta}{sin\theta}.$$

Definition 4. **Fundamental Identity:**

$$sin^2\theta + cos^2\theta = 1.$$

Example 2. *If a right triangle* ABC*, with angle* C *being the right angle, has* $cscB = \frac{7}{4}$*, then find* $cosB$*.*

Solution: *Since* $cscB = \frac{hypotenuse}{opposite}$*, then* $hypotenuse = 7$ *and* $opposite = 4$*. Then use the Pythagorean theorem to find the length of the adjacent side of* B*.*

$$(opposite)^2 + (adjacent)^2 = (hypotenuse)^2$$
$$4^2 + (adjacent)^2 = 7^2.$$

Then $adjacent = \sqrt{33}$*. Hence,* $cosB = \frac{adjacent}{hypotenuse} = \frac{\sqrt{33}}{7}$*.*

Definition 5. **Trigonometric Function Values of Common Angles:**

θ	$sin\theta$	$cos\theta$	$tan\theta$	$csc\theta$	$sec\theta$	$cot\theta$
30°	$\frac{1}{2}$	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{3}}{3}$	2	$\frac{2\sqrt{3}}{3}$	$\sqrt{3}$
45°	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{2}}{2}$	1	$\sqrt{2}$	$\sqrt{2}$	1
60°	$\frac{\sqrt{3}}{2}$	$\frac{1}{2}$	$\sqrt{3}$	$\frac{2\sqrt{3}}{3}$	2	$\frac{\sqrt{3}}{3}$